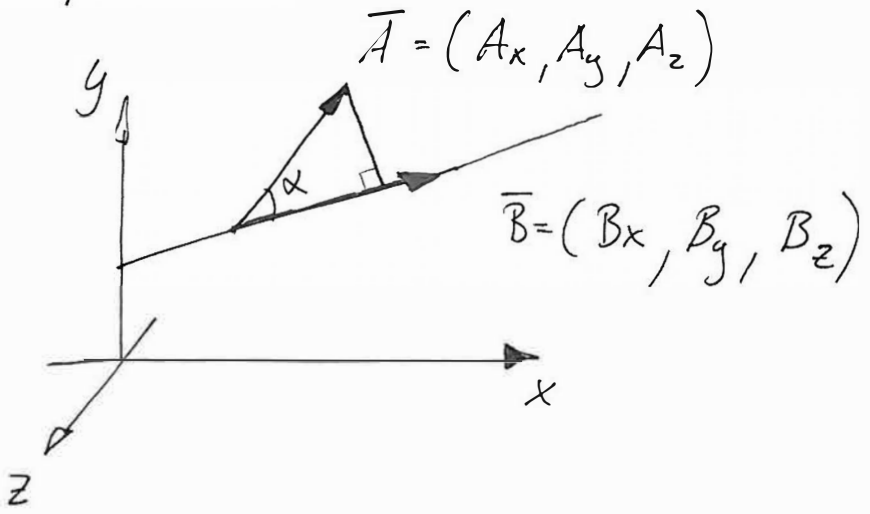
Projektionen av en vektor



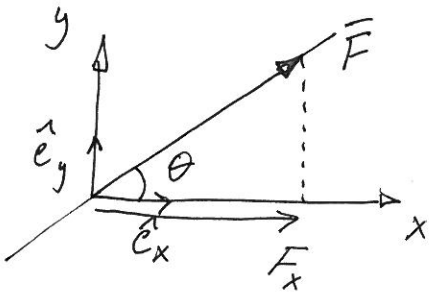
Skalär multiplikation av vektorer

$$\bar{A} \cdot \bar{B} = (A_x, A_y, A_z) \cdot (B_x, B_y, B_z)$$

$$= A_x \cdot B_x + A_y \cdot B_y + A_z \cdot B_z$$

$$= |\bar{A}| \cdot |\bar{B}| \cos\alpha = A \cdot B \cos\alpha$$

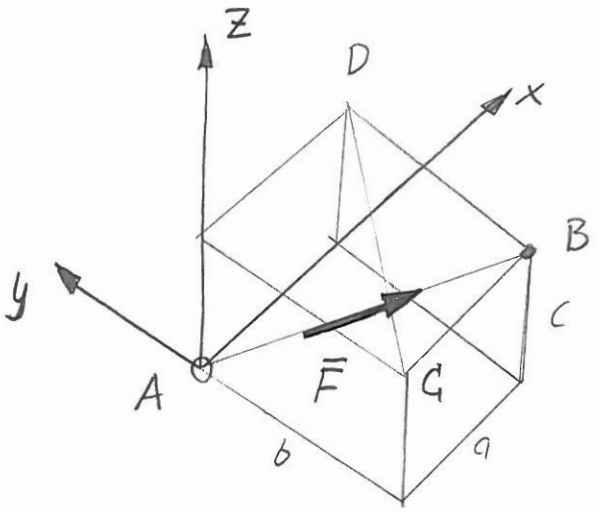
Räkneregel



$$F_x = \bar{F} \cdot \hat{e}_x = |\bar{F}| \cdot |\hat{e}_x| \cdot \cos\theta = F \cdot 1 \cdot \cos\theta$$

$$\bar{F}_x = F_x \cdot \hat{e}_x = \underbrace{\bar{F} \cdot \hat{e}_x}_{\text{skalar}} \cdot \underbrace{\hat{e}_x}_{\text{vektor}}$$

Ex)



$F = |\bar{F}|$ storlek

Bestäm kraftens projektion på DC (storlek)

$$\hat{e}_{DC} = \frac{\overline{DC}}{|\overline{DC}|} = \frac{(-a, -b, 0)}{\sqrt{a^2+b^2+0^2}}$$

$$\bar{F} = F \cdot \hat{e}_{AB} = F \cdot \frac{\overline{AB}}{|\overline{AB}|} = \frac{F \cdot (a, -b, c)}{\sqrt{a^2+b^2+c^2}}$$

$$F_{DC} = \bar{F} \cdot \hat{e}_{DC} = \frac{F \cdot (a, -b, c)}{\sqrt{a^2+b^2+c^2}} \cdot \frac{(-a, -b, 0)}{\sqrt{a^2+b^2}}$$

$$= \frac{F\left(a \cdot (-a) + (-b) \cdot (-b) + c \cdot 0\right)}{\sqrt{a^2+b^2+c^2}\ \sqrt{a^2+b^2}} = \frac{F\left(-a^2+b^2\right)}{\sqrt{a^2+b^2+c^2}\ \sqrt{a^2+b^2}}$$

Jämvikt i 3 dimensioner

Jämviktsvillkor

$$\boxed{\begin{aligned}\Sigma \bar{F} &= \bar{0} \\ \Sigma \bar{M}_A &= 0\end{aligned}}$$

$\Sigma F_x = 0 \quad \Sigma F_y = 0 \quad \Sigma F_z = 0$

$\Sigma M_x = 0 \quad \Sigma M_y = 0 \quad \Sigma M_z = 0$

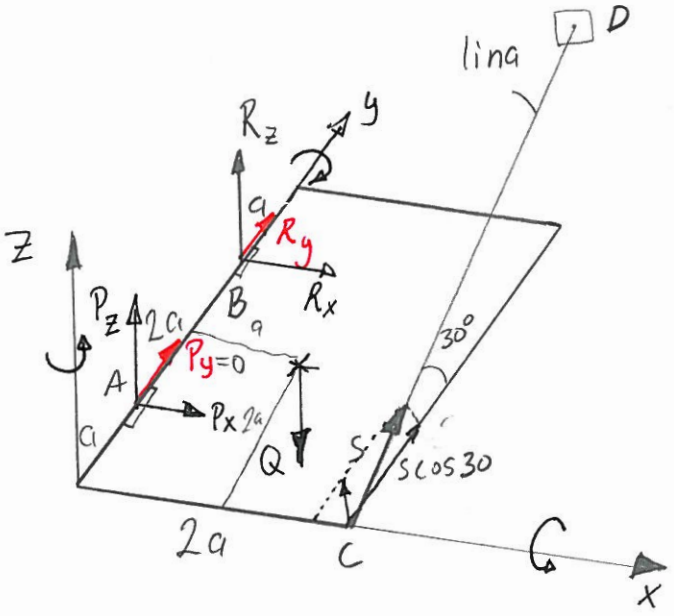
6 ekvationer.

$$\bar{F} = (F_x, F_y, F_z) = F_x \cdot \hat{e}_x + F_y \cdot \hat{e}_y + F_z \cdot \hat{e}_z$$

$$\bar{M} = (M_x, M_y, M_z) = M_x \cdot \hat{e}_x + M_y \cdot \hat{e}_y + M_z \cdot \hat{e}_z$$

Frilägganingssymboler sid 61, 62 i läroboken.

Ex



A o B är gångjärn

$\begin{cases} 7 \text{ obekanta ekvationer} \\ 6 \text{ ekvationer} \\ \text{sätt } P_y = 0 \end{cases}$

Givet Q = luckans tyngd

$\sin 30° = \frac{1}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$

Sök S + komp av P o R

Bestäm kraften i Linan

$\boxed{\Sigma \bar{F} = 0}$

$\Sigma F_x = 0 \Rightarrow P_x + R_x = 0 \quad (1)$

$\Sigma F_y = 0 \Rightarrow \underbrace{P_y}_{0=} + R_y + S\cdot\cos 30° = 0 \quad (2)$

$\Sigma F_z = 0 \Rightarrow P_z + R_z + S\cdot\sin 30 - Q = 0 \quad (3)$

$\boxed{\Sigma \bar{M} = 0}$

$\Sigma M_x = 0 \Rightarrow P_z \cdot a + R_z \cdot 3a - Q \cdot 2a = 0 \quad (4)$

$\Sigma M_y = 0 \Rightarrow Q \cdot a - S \cdot \sin 30° \cdot 2a = 0 \quad (5)$

$\Sigma M_z = 0 \Rightarrow S \cdot \cos 30° \cdot 2a - P_x \cdot a - R_x \cdot 3a = 0 \quad (6)$

(5) → $S = Q$

(2) → $R_y = -\frac{\sqrt{3}}{2} Q$

(1) → $P_x = -R_x$ ins i (6) ⇒

$Q \cdot \frac{\sqrt{3}}{2} \cdot 2a + R_x \cdot a - R_x \cdot 3a = 0$

$2R_x = \sqrt{3} \cdot Q \Rightarrow \boxed{R_x = \frac{\sqrt{3}}{2} Q} \quad \boxed{P_x = -\frac{\sqrt{3}}{2} Q}$

(4) $\Rightarrow$ $P_z = 2Q - 3R_z$ ins i (3)

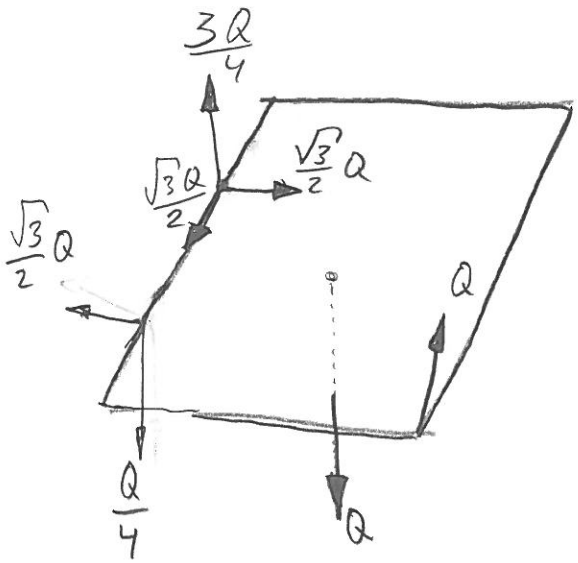
$$2Q - 3R_z + R_z - \frac{Q}{2} = 0$$

$$-2R_z = -\frac{3}{2}Q$$

$$R_z = \frac{3}{4}Q$$

$$P_z = 2Q - \frac{3 \cdot 3}{4}Q = \frac{8Q}{4} - \frac{9Q}{4} = -\frac{1}{4}Q$$

Verklig bild



$$P_x = -\frac{\sqrt{3}}{2}Q$$

$P_y = 0$ villkor

$$P_z = -\frac{1}{4}Q$$

$$R_x = \frac{\sqrt{3}}{2}Q$$

$$R_y = -\frac{\sqrt{3}}{2}Q$$

$$R_z = \frac{3}{4}Q$$

$S = Q$